BEDLOAD TRANSPORT DUE TO ASYMMETRIC AND SKEWED WAVES PLUS A CURRENT

David Gonzalez-Rodriguez and Ole Secher Madsen[1]

Prediction of nearshore sediment transport is a fundamental, open problem in coastal engineering. In this paper, we extend an existing model to predict bedload due to pure asymmetric and skewed waves to the case of combined waves and currents. The choice of the appropriate bed roughness to compute bedload is discussed. Predictions based on different choices of roughness are compared with available experimental data. Depending on whether a current is present or absent, agreement between predictions and measurements is obtained for a different choice of the roughness. Rather than using a different roughness parameterization for the two cases, these results suggest the necessity of revising the model that predicts the bed shear stress used in the calculation of bedload transport.

INTRODUCTION

Cross-shore sediment transport in the nearshore region, of crucial importance to coastal engineers, is due to the simultaneous effect of waves and currents. Nearshore waves are both asymmetric (forward-leaning in shape) and skewed (with peaked, narrow crests and wide, flat troughs). Existing models to predict nearshore sediment transport are not entirely satisfactory, since they are either highly empirical or require intensive numerical computations. Furthermore, most of the previous studies focused on skewed waves, while little attention has been paid to the effect of wave asymmetry. The authors developed a simple model capable of predicting bedload transport due to asymmetric and skewed pure waves (Gonzalez-Rodriguez and Madsen 2007, hereafter referred to as GRM07). In this paper we discuss the extension of the model to the case of combined waves and currents.

ASYMMETRIC AND SKEWED WAVES ALONE

GRM07's model uses Madsen's (1991) bedload transport formula to express the instantaneous transport as a function of the instantaneous bed shear stress, $\tau_b(t)$. Madsen's formula is only applicable when suspension effects are negligible, i.e., when the ratio of the maximum shear velocity (u_{*m}) to the sediment fall velocity (w_s) is smaller than a certain threshold value ($u_{*m}/w_s < 2.7$ was suggested by GRM07 for pure waves). To account for the effects of wave shape, the instantaneous bed shear stress is related to the near-bed orbital velocity, $u_b(t)$, through

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$$\tau_b(t) = \frac{1}{2}\rho f_w(t) | u_b(t + l_\tau(t)) | u_b(t + l_\tau(t)) , \tag{1}$$

where l_τ is the time lag between the bed shear stress and the near-bed velocity, and f_w is a generalized, time-dependent friction factor. Specifically, the values of $f_w(t)$ and $l_\tau(t)$ at the wave crest are taken as those obtained for a sinusoidal wave of velocity amplitude equal to the crest velocity and quarter-period equal to the interval between the zero up-crossing and the maximum velocity. The values at the wave trough are calculated analogously, and the time-dependencies of $f_w(t)$ and $l_\tau(t)$ over the wave period are defined as linear interpolations between their values at crest and trough. A similar parameterization, using different friction factors at crest and trough but without a continuous time variation was proposed by Silva et al. (2006). This simple model accounts for the increased onshore bed shear stress in a purely asymmetric wave, in which the boundary layer associated with the onshore motion has a shorter time to develop than that associated with the offshore motion. As shown by GRM07, the model's predictions agree with measurements of sheet flow bedload due to both asymmetric and skewed pure wave motion when the bed roughness is taken equal to the mean sediment diameter, k_N=D_{50}.

As shown in several sheet flow studies, the total hydraulic roughness that parameterizes the near-bed velocity is larger than the sediment diameter. For example, Herrmann and Madsen (2007) obtained the following empirical expression for the total sheet flow roughness,

$$k_N = \left[4.5\left(\Psi - \Psi_{cr}\right) + 1.7\right] D_n , \tag{2}$$

based on limited experimental data, where Ψ and Ψ_{cr} are the Shields parameter and its critical value for initiation of motion, respectively, and D_n is the nominal diameter (D_n≈1.1D_{50}). Even if the hydraulic sheet flow roughness is parameterized by the total mobile-bed roughness roughness, this is not necessarily the appropriate value of the roughness to use when computing the bed shear stress responsible for transport. In fact, accurate predictions of transport over rippled beds have been obtained by using k_N=D_{50}, instead of the total hydraulic roughness (Madsen and Grant, 1976). For this reason, and based on the good agreement with experimental results, GRM07 used k_N=D_{50} to successfully predict sheet flow bedload under asymmetric and skewed pure waves (see figures 9 and 10 in GRM07).

In contrast, Figs. 1 and 2 show poor agreement between GRM07's model predictions and measurements when bedload is based on the mobile-bed roughness (Eq. 2). Figs. 1 and 2 include the same experimental cases as figures 9 and 10 in GRM07, respectively. Fig. 1 shows a comparison between predicted and measured net transport rates under skewed waves. For the data shown in the figure, the mobile-bed roughness is of the order of (10-30)D_{50}, and the model overpredicts the measurements by a factor of about 4.3. Fig. 2 shows a comparison between predicted and measured net transport rates under asymmetric waves. For King's (1991) asymmetric wave cases, the mobile-bed roughness, about $4D_{50}$, is of the same order as the grain diameter, and the use of either roughness does not significantly affect the model's predictions. In contrast, for Watanabe and Sato's (2004) cases, the mobile-bed roughness is significantly larger than the grain diameter (by a factor of about 17), and the predictions of the model improve dramatically for this data set when the mobile-bed roughness is used. It is noted that the value of u_{*m}/w_s for a specific case increases when the sand-grain roughness is replaced by the mobile-bed roughness. Consequently, the numerical threshold above which suspension effects become appreciable (u_{*m}/w_s=2.7 in GRM07, which was determined based on k_N=D_{50}) depends on the choice of roughness. Instead of defining a new threshold for the mobile-bed roughness, the data points included in Figs. 1 and 2 are those that met the original threshold in GRM07, although the new values of u_{*m}/w_s for some of these points are now larger than 2.7.

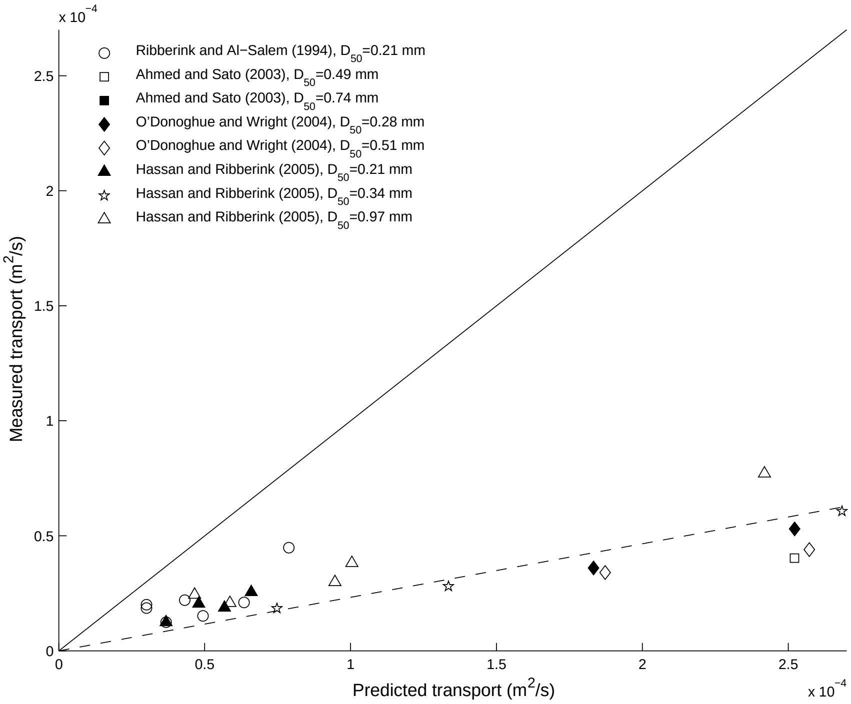

Figure 1. Comparison between predicted and measured net sediment transport rates for skewed, symmetric waves (no current). Predicted bedload is based on mobile-bed roughness. Two datapoints, for which the transport rate is overpredicted by factors of 5.8 and 7.7, fall outside the depicted range. The solid line corresponds to perfect agreement between predictions and measurement, while the dashed line is the least-squares fit to the data and corresponds to an overprediction by a factor of 4.3.

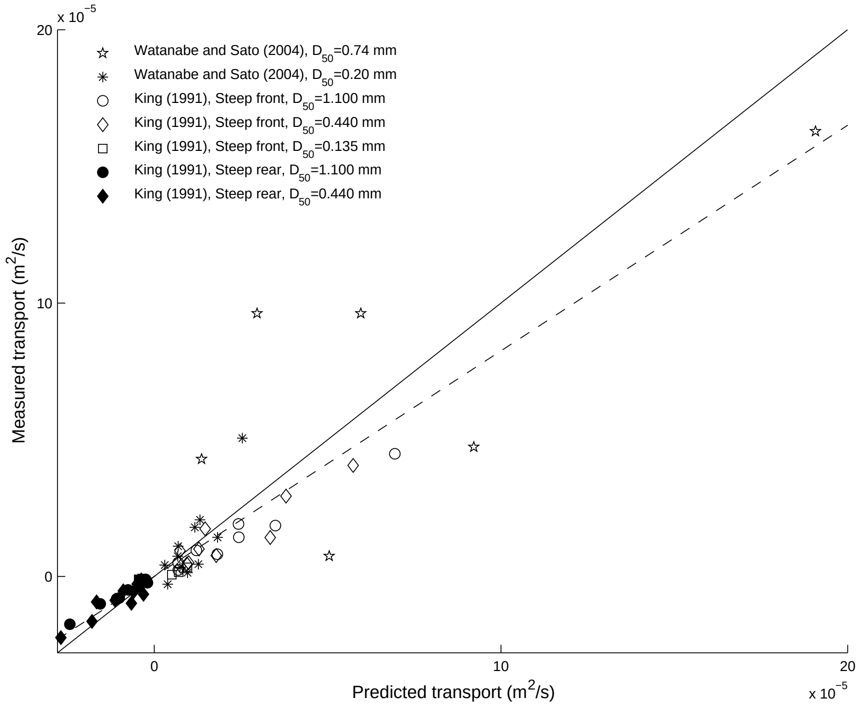

Figure 2. Comparison between predicted and measured net sediment transport rates for asymmetric, non-skewed waves (no current). Predicted bedload is based on mobile-bed roughness. The solid line corresponds to perfect agreement between predictions and measurement, while the dashed line is the least-squares fit to the data and corresponds to an overprediction by a factor of 1.2.

SINUSOIDAL WAVES PLUS A CURRENT

To predict bed shear stress due to combined sinusoidal waves plus a current, we apply the Grant and Madsen boundary layer model as presented by Madsen (1994) with the modification suggested by Madsen and Salles (1998). The model is applied to sheet flow experimental conditions, and therefore the hydraulic roughness is taken equal to the mobile-bed roughness (Eq. 2). As discussed in the previous section, two possible choices of bedload roughness are considered in order to obtain the bed shear stress responsible for sediment transport: $k_N = D_{50}$ and the mobile-bed roughness. In the former case, the wave-current model is applied as follows (details are presented in the Appendix). First, a wave-current analysis based on mobile-bed roughness is performed, using the reference current velocity measured at a reference level. From this first analysis, the wave-current boundary layer thickness, δ_{cw}, and the current velocity at $z = \delta_{cw}$ are determined. This current velocity is used as a new reference velocity for a second wave-current analysis, now based on $k_N=D_{50}$. From this second analysis, the (skin friction) bed shear stress is determined and used in Madsen's (1991) formula for the bedload computations.

Figs. 3 and 4 show comparisons of net transport rates by our conceptual model and oscillatory wave tunnel measurements for sinusoidal waves plus currents by Dohmen-Janssen et al. (2002, series E, I, J). The figure only includes measurements for which appreciable suspension effects are not expected. By analogy with GRM07, the threshold of appreciable suspension is established at u_{*m}/w_s=2.7, where u_{*m} is now the maximum combined wave-current shear velocity based on $k_N=D_{50}$. In Fig. 3, the predicted bed shear stress used in the bedload calculations is based on $k_N=D_{50}$, which yields an underprediction of the measurements by a factor of about 4.6. In Fig. 4, the predicted bed shear stress used in computations of the net bedload transport rates is based on the mobile-bed roughness, which is of the order of $14D_{50}$ for these experimental conditions. This choice yields an excellent agreement between predicted and measured transport rates.

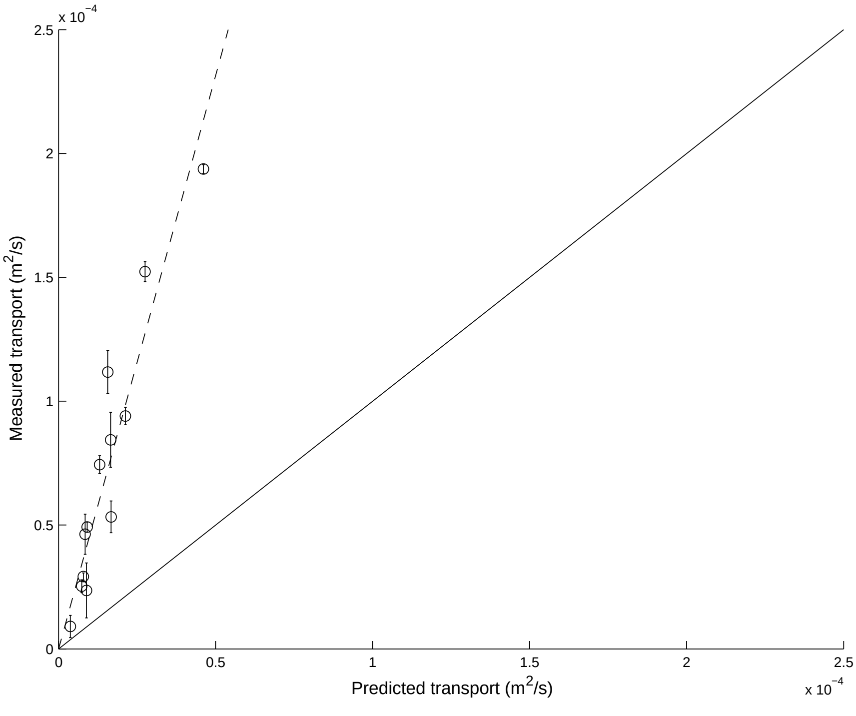

Figure 3. Comparison between predicted and measured (Dohmen-Janssen et al., 2002) net sediment transport rates in current direction for co-directional sinusoidal waves and currents. Predicted bedload is based on $k_N=D_{50}$. The line of perfect

agreement is shown. The solid line corresponds to perfect agreement between predictions and measurement, while the dashed line is the least-squares fit to the data and corresponds to an underprediction by a factor of 4.6.

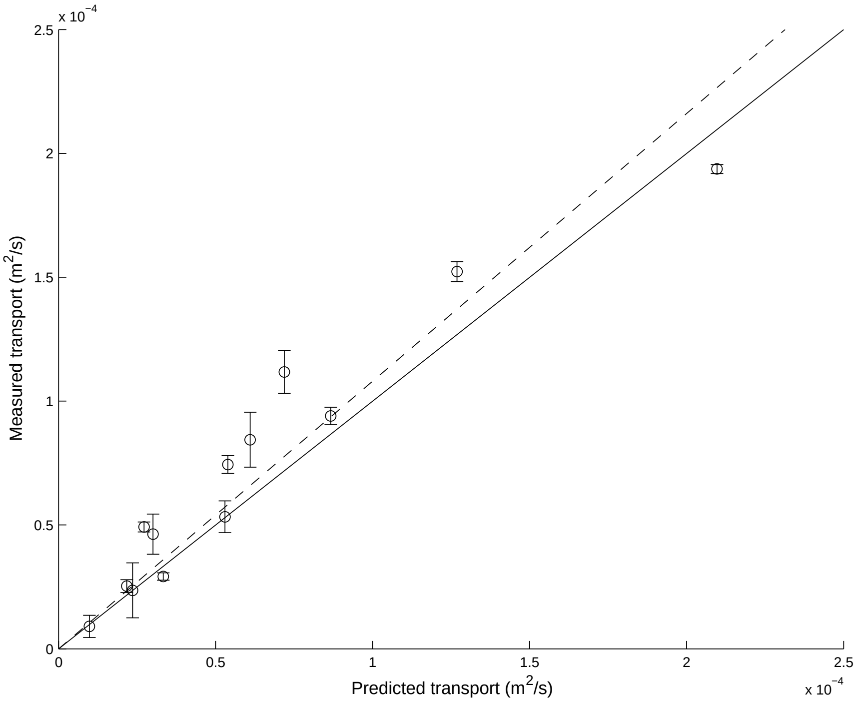

Figure 4. Comparison between predicted and measured (Dohmen-Janssen et al., 2002) net sediment transport rates in current direction for co-directional sinusoidal waves and currents. Predicted bedload is based on mobile-bed roughness. The solid line corresponds to perfect agreement between predictions and measurement, while the dashed line is the least-squares fit to the data and corresponds to an underprediction by a factor of 1.1.

ASYMMETRIC AND SKEWED WAVES PLUS A CURRENT

To compute bedload under asymmetric and skewed waves plus a current, the total bed shear stress is decomposed into the sum of the wave and the current shear stresses,

$$\tau_b(t) = \tau_{wb}(t) + \tau_{cb} \,. \tag{3}$$

To calculate the current shear stress, τ_{cb}, the waves are approximated by an equivalent sinusoidal wave with the same period as the original wave and velocity amplitude $u_{bm}=(u_c-u_t)/2$, where u_c and u_t are the crest and trough velocities of the original wave, respectively. With the equivalent periodic wave specified in this manner, the sinusoidal wave-current analysis described in the previous section is applied to obtain τ_{cb}. Using this current shear stress, the wave-current friction factor is calculated at crest and trough accounting for wave asymmetry and skewness, as previously outlined for pure waves, but now adding the effect of the current. Analogous to the pure wave case, a time-varying wave-current friction factor is obtained by linear interpolation between the crest and trough values, from which the wave shear stress $\tau_{wb}(t)$ is determined. The relevant expressions involved in these calculations are summarized in the Appendix. Due to the lack of bedload laboratory data with significant contributions to the transport from both the current and the wave asymmetry and/or skewness, comparison of this proposed methodology for the computation of bedload transport rates in combined skewed and asymmetric wave and current flows with measurements is not presented.

CONCLUSION

We have presented an extension of our bedload model to the case of combined waves and currents and discussed the choice of bed roughness. The appropriate bed roughness to parameterize the bed shear stress responsible for bedload sediment transport remains unclear. For purely skewed waves, good agreement with experimental data is obtained when using a roughness equal to the grain diameter, while the use of the total mobile-bed roughness yields overpredictions by a factor of about 4.3. In contrast, for sinusoidal waves plus a current, good agreement with the available data is obtained by using the total mobile-bed roughness, while the use of the grain diameter yields underpredictions by a factor of about 4.6. Similarly, the use of the mobile-bed roughness yields a dramatically improved agreement with some of the pure asymmetric wave data. For the remaining asymmetric wave data, for which the mobile-bed roughness is of the same order as the grain diameter, the agreement between predictions and measurements is good for either choice of the roughness.

The inconsistent choice of roughness needed to predict bedload in cases with and without currents may be avoided by modifying the model used to predict the bed shear stress. GRM07 presented comparisons between bed shear stresses predicted by their simple conceptual model, also used here, and by a numerical k-ε model. The predictions of both models showed good agreement for purely asymmetric waves, while the conceptual model slightly overpredicted the onshore bed shear stresses for skewed waves. This overprediction may be responsible for the overprediction of bedload due to skewed waves when the mobile-bed roughness is used. The authors are currently working on a more rigorous model for the prediction of the bed shear stress under skewed and asymmetric waves, which is anticipated to resolve the inconsistency of having to use a bed roughness that depends on whether a current is present or not when computing the bed shear stress responsible for bedload sediment transport.

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APPENDIX

Following Madsen (1994), the bed shear stress due to the waves is

$$\tau_{wb}(t) = \tfrac{1}{2}\rho f_{cw} \left|u_b(t)\right| u_b(t) , \tag{A-1}$$

where f_{cw}, the combined bed-current shear stress, is given by

$$f_{cw} = \begin{cases} C_\mu(7.0X^{-0.078} - 8.8) & \text{for } \ 0.2 < X < 10^2 \\ C_\mu(5.6X^{-0.109} - 7.3) & \text{for } \ 10^2 < X < 10^4, \end{cases} \tag{A-2}$$

with $X=(C_\mu u_{bm})/(k_N \omega)$ and

$$C_\mu = \sqrt{1 + 2\left|\cos\varphi_{cw}\right| \frac{\tau_{cb}}{\tau_{wm}} + \left(\frac{\tau_{cb}}{\tau_{wm}}\right)^2} \ , \tag{A-3}$$

where φ_{cw} is the angle between the current and the direction of wave propagation, τ_{cb} is the modulus of the current shear stress, and $\tau_{wm} = \rho\, u_{*wm}^2 = ½\, \rho\, f_{cw}\, u_{bm}^2$ is the maximum shear stress due to the waves. For an asymmetric or skewed wave, f_{cw} and τ_{wm} adopt different values at the crest and at the trough, since they are calculated based on u_c and u_t and the corresponding quarter-periods.

The bed shear stress due to the current is calculated as

$$\tau_{cb} = \rho u_{*c}^{\ 2} , \tag{A-4}$$

where u_{*c}, the current shear velocity, is related to the current velocity at a height z_r above the bed, $u_{c,r}$,

$$u_{c,r} = \frac{u_{*c}}{\kappa}\left(\ln\frac{z_r}{\delta_{wc}} + \frac{u_{*c}}{u_{*m}} \ln\frac{\delta_{wc}}{z_0} \right), \tag{A-5}$$

where κ =0.4, $z_0 = k_N/30$, and

$$\delta_{wc} = \frac{\kappa u_{*m}}{\omega} \exp\left(2.96X^{-0.071} - 1.45\right). \tag{A-6}$$

with the exponential scaling factor being the modification of Madsen (1994) suggested by the results obtained by Madsen and Salles (1998). The wave-current shear velocity, u_{*m}, is given by

$$u_{*m}^{\ 2} = C_\mu u_{*wm}^{\ 2} . \tag{A-7}$$

X and u_{*wm} are calculated using the expressions for the wave shear stress above, based on *equivalent periodic* wave characteristics (total period T and velocity amplitude $u_{bm} = (u_c - u_t)/2$) if the waves are not sinusoidal.

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